Wavelet-Based Image Compression and Denoising

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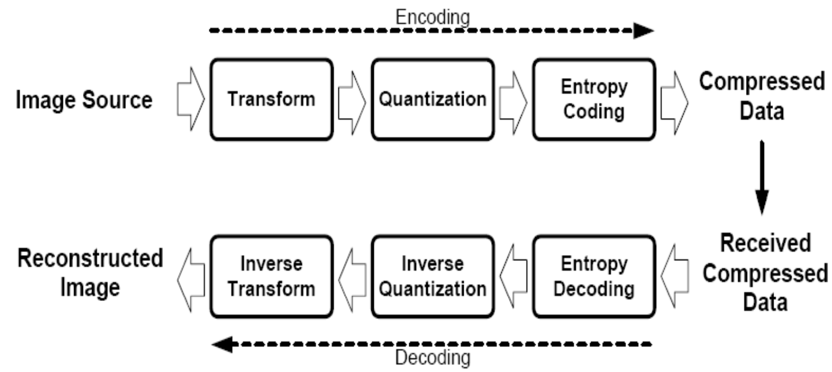
Diagram for Wavelet-based Image Processing/Compression



Wavelet Filters in Image Processing

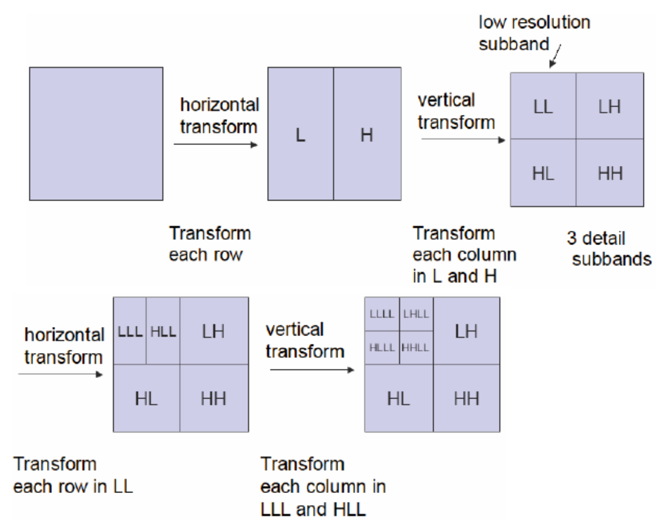
- LeGall 5/3 Filter:
  $$h = \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right], \quad \tilde{h} = \left[-\frac{1}{8}, \frac{3}{4}, \frac{1}{4}, \frac{3}{4}, -\frac{1}{8}\right].$$
- $H(z) = z(1+z^{-1})^2/4, \quad \tilde{H}(z) = z(1+z^{-1})^2(4-z-z^{-1})/8.$
- Daubechies 9/7 filter:

$$H(z) = z^2(1+z^{-1})^4(\rho - z - z^{-1})/(16\rho - 32).$$

$$\tilde{H}(z) = z^2(1+z^{-1})^4\left(z^2 + z^{-2} - (8-\rho)(z+z^{-1}) + \frac{128}{5\rho} + 2\right)\frac{1}{64(64/5\rho - 6 + \rho)}$$

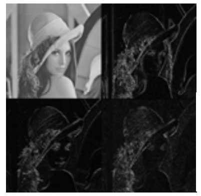
$$\rho = \frac{8}{3} + \frac{2}{3}\sqrt[3]{\frac{7}{25}}\left(\sqrt[3]{10+3\sqrt{15}} + \sqrt[3]{10-3\sqrt{5}}\right)$$

2D Tensor-product DWT Using 1D DWT

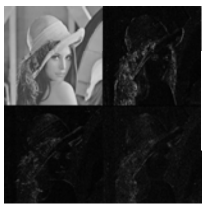


An Example

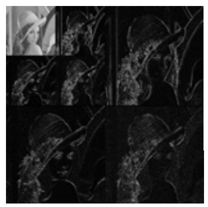
Original



1 level Haar

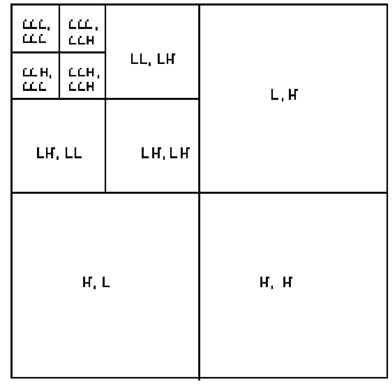


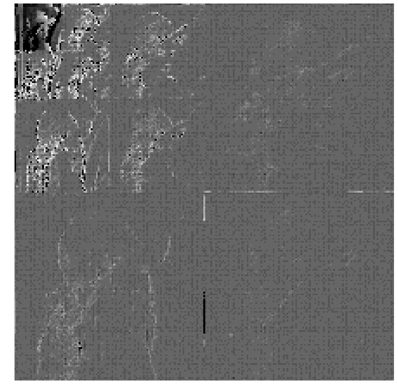
1 level linear spline



2 level Haar

Tree Structure of Wavelet Coefficients

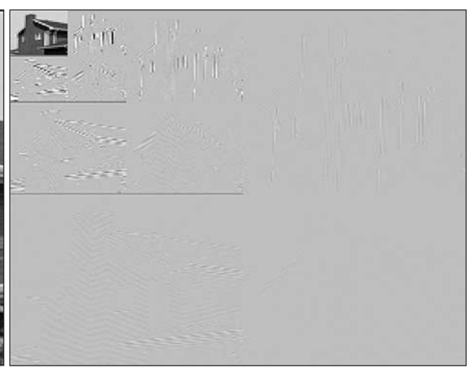




Most highpass wavelet coefficients are negligible (small)

Another Example

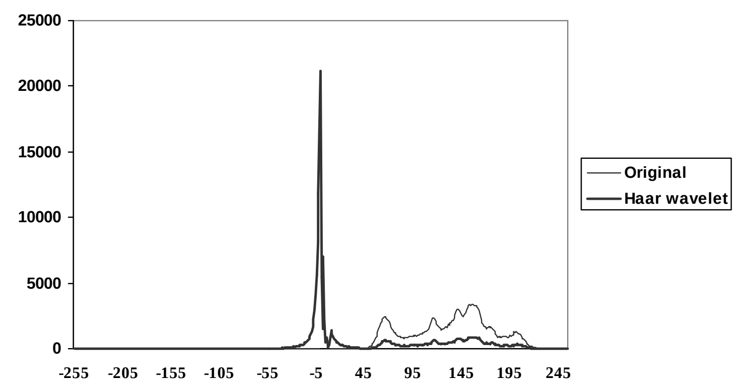
▲ 2. Original image used for demonstrating the 2-D wavelet transform.



▲ 5. A three-level ($K = 3$), 2-D wavelet transform using the symmetric wavelet transform with the 9/7 Daubechies coefficients (the high-frequency bands have been enhanced to show detail).

Sparse Wavelet Representation

- Wavelet coefficient histogram

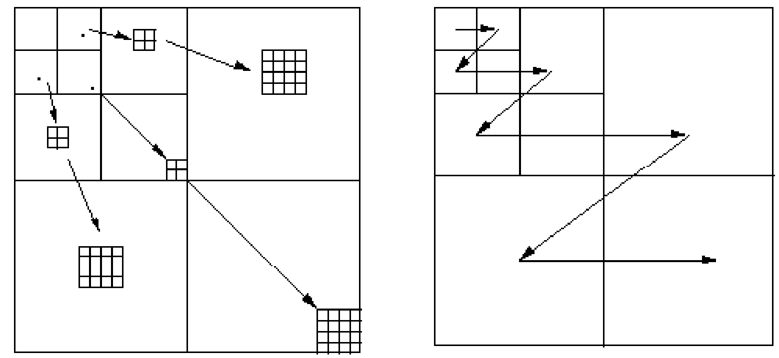


Small Entropy Numbers

- Coefficient entropies

	Entropy
Original image	7.22
1-level Haar wavelet	5.96
1-level linear spline wavelet	5.53
2-level Haar wavelet	5.02
2-level linear spline wavelet	4.57

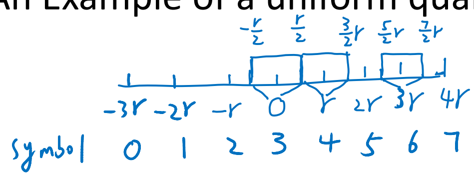
Correlation Between Different Scales



- There are redundancy among bands (frequency)
- Coefficients in bands are from the same spatial place.

Quantization Schemes

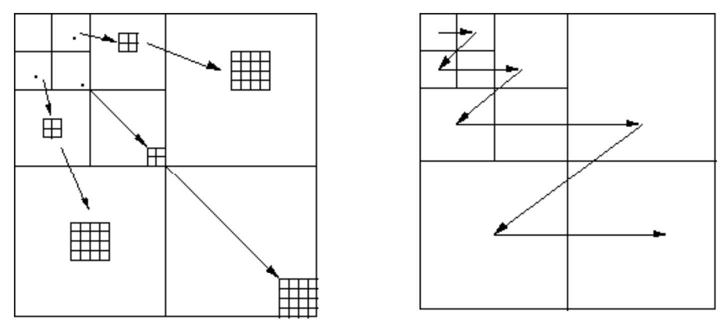
- Scalar Quantization: Each real number is quantized separately.
- An Example of a uniform quantizer:



If $-\frac{r}{2} \le x < \frac{r}{2}$, replace x by 0.

- Neighbouring coefficients are grouped into a vector. A code book is used to quantize it.

EZW (Embedded Zerotree Wavelet)

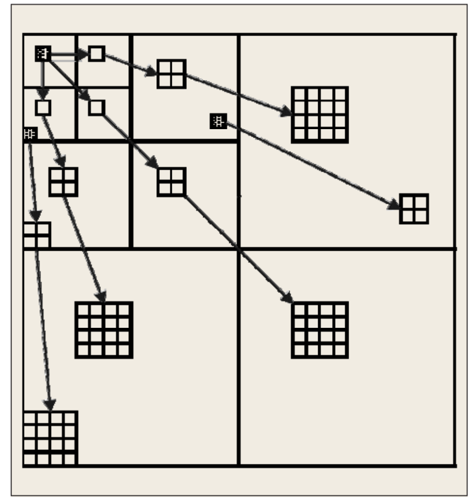


Hypothesis of EZW: If a wavelet coefficient at a coarse scale is insignificant with respect to a given threshold T, then all wavelet coefficients of the same orientation in the same special location at a finer scales are likely to be insignificant with respect to T.

SPIHT (Modified EZW)

- Using a similar idea, the EZW has been further improved by considering Set Partitioning in Hierarchical Trees (SPIHT).
- For EZW, see [J. M. Sapiro, Embedded Image Coding Using Zerotrees of Wavelet Coefficients, IEEE Trans. SP, 41 (1993), 3445-3462].
- For SPIHT, see [A. Said and W.A. Pearlman, A New, fast, and Efficient Image Codec Based on Set Partitioning in Hierarchical Trees, IEEE Trans. CSVT, 6 (1996), 243-250]
- Both EZW and SPHIT are progressive coder (can terminated at any desired bit stream).

Significant Map Coding Using Zerotree



Four types of Label
1.Positive significant
2.Negative significant
3.Isolated zero
4.Zero tree root

For each coefficient:
Give a label based on predefine threshold T

$$T_0 = 2^{\lfloor \log_2 x_{\max} \rfloor}$$

Basics on Coding Schemes

- Code numbers and characters in ASCII (American Standard Code for Information Interchange) and in UNICODE (used by Java)
- Shannon entropy: {x1, ..., xn} with probability p: $-\sum_{j=1}^{n} p(x_j)\log p(x_j)$
- Entropy-based coding schemes: Huffmann coding and arithmetic coding.
- Symbols with high probability are encoded with less number of bits

An Example of Coding Schemes

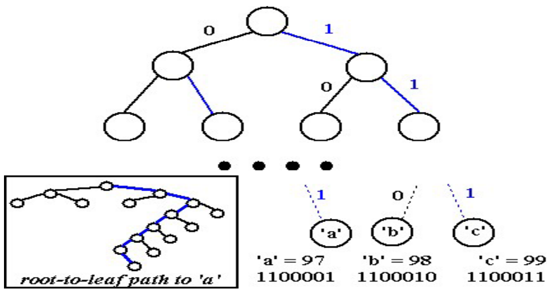
- An example to code: go go gophers
- In ASCII (8-bit), 3-bit, and Huffman coding

coding a message

ASCII coding			3-bit coding		
char	ASCII	binary	char	code	binary
g	103	1100111	g	0	000
o	111	1101111	o	1	001
p	112	1110000	p	2	010
h	104	1101000	h	3	011
e	101	1100101	e	4	100
r	114	1110010	r	5	101
s	115	1110011	s	6	110
space	32	1000000	space	7	111

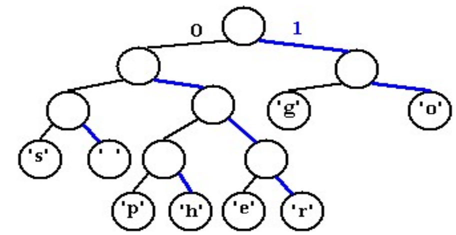
Example and Coding Trees

- go go gophers in ASCII is coded as:
- 103 111 32 103 111 32 103 111 112 104 101 114 or 115
  1100111 1101111 1100000 1100111 1101111 1000000 1100111 1101111 1110000 1101000 1100101 1110010 1110011
- 3-bit: 0 1 7 0 1 7 0 1 2 3 4 5 6 or 000 001 111 000 001 111 000 001 010 011 100 101 110 111
- Coding tree for ASCII:



Coding Tree

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



Under this coding tree, gophers is encoded as

10 11 001 10 11 001 10 11 0100 0101 0110 0111 000

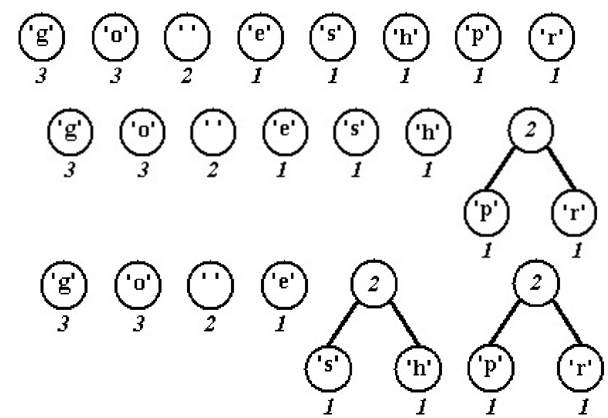
Coding tree can be used to decode the bit stream

Building Huffman Coding 1

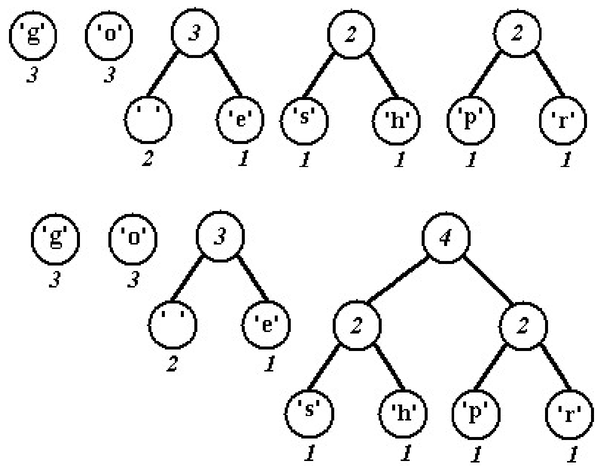
- Begin with a forest of trees. All trees have one node with weights.
- Combine two trees with smallest weights.
- An optimal encoding tree is the last single tree

Huffman Coding: 2

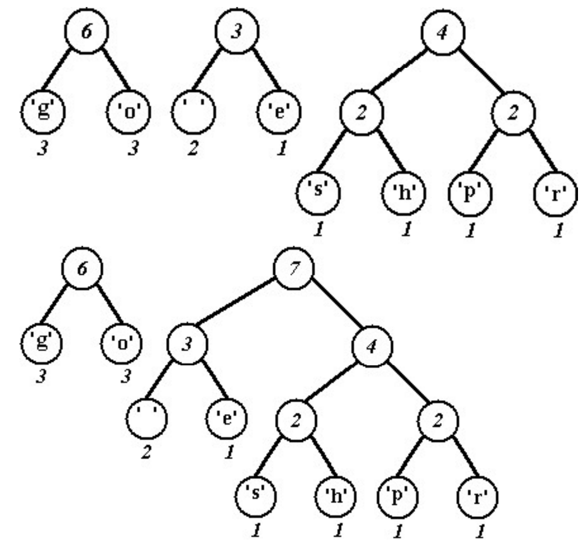
- Build a Huffman coding tree for symbols: go go gophers



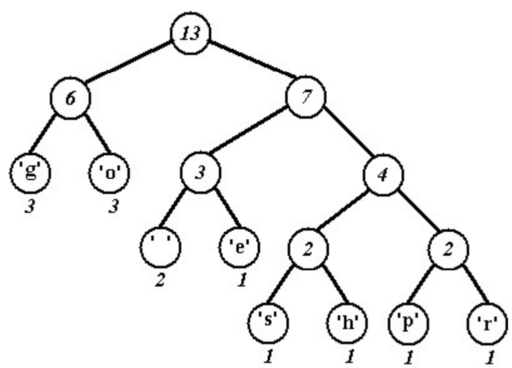
Continuing



Continuing



Final Huffman Coding Tree



- go go gophers can be encoded as
  00 01 100 00 01 100 00 01 1110 1101 101 1111 1100

Compression by Huffman Coding

- The file contains: the Huffman coding tree or information how to build it
- The bit stream for the coded message
- The end of bit stream symbol (EOF), since it is not possible to store 1 bit in a computer.

Arithmetic Coding

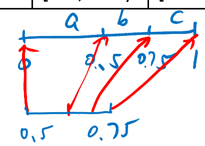
- No tree is needed. The length of the subinterval is proportional to probablity
- An example to code baca: a (2*), b(1*), c(1*)

symbol	a	b	c
Frequency(probability)	2	1	1
Interval	[0, 0.5)	[0.5,0.75)	[0.75,1)

b 0,5 0,75

a 0,5 0,625

c 0,59375 0.625



a 0,59375 0.6375 output

Measuring Quality: MSE and PNSR

- Let I be the original image and I' be the reconstructed image of size M*N.
- MSE=Mean Square Error is

$$\mathrm{MSE} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}|I(m,n) - I'(m,n)|^2$$

- PNSR=Peak Signal to Noise Ration is

$$PSNR = 10\log_{10}\frac{255^2}{MSE}$$

- The smaller MSE or large PSNR, the closer I' to I.

Image compression results

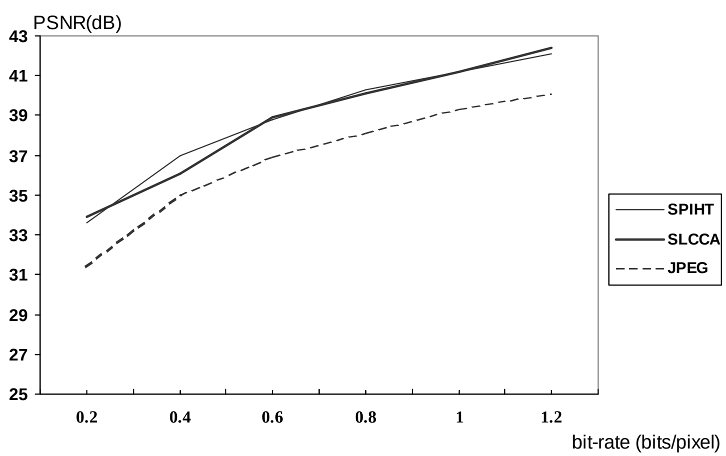


Image compression results

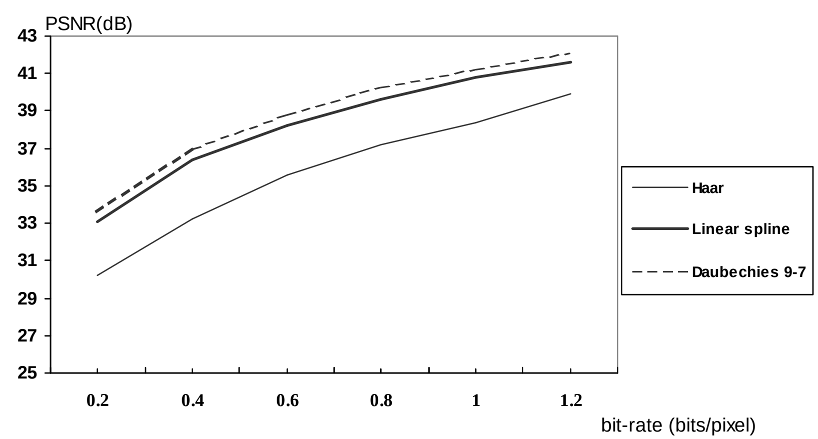


Image compression results

SPIHT 0.2 bits/pixel

JPEG 0.2 bits/pixel

Image compression results

SPIHT

JPEG

Wavelet-based Image Denoising

- Image denoising: Removing unwanted noise in order to restore the original image.
- Wavelet transform provides us with one of the methods for image denoising.
- Wavelet transform, due to its excellent localization property, has rapidly become an indispensable signal and image processing tool for a variety of applications, including denoising and compression.
- Wavelet denoising attempts to remove the noise present in the signal while preserving the signal characteristics, regardless of its frequency content.

Steps for Image Denoising

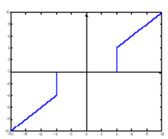
- Wavelet-based denoising involves three steps:
  - a forward DWT
  - nonlinear thresholding step
  - an inverse DWT
- Methods used for thresholding in Step 2:
  - Universal Thresholding
  - Visu Shrink
  - Sure Shrink
  - Bayes Shrink
- Key idea: removes noise by killing coefficients that are insignificant relative to some threshold.

Hard and Soft Thresholding

- The hard thresholding operator is defined as

  D(x, λ) = x for all |x|> λ

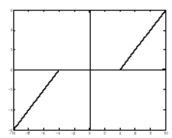
- Hard threshold is a "keep or kill" procedure and is more intuitively appealing.
- The transfer function of the same is shown here.



- The soft thresholding operator is defined as

  D(x, λ) = sgn(U)max(0, |x| - λ)

- Soft thresholding shrinks coefficients above the threshold in absolute value.
- The transfer function of the same is shown here.



Universal or Global Thresholding

- The threshold

$$\lambda_{UNIV} = \sqrt{2\ln N}\sigma$$

  (N being the signal length, σ being the noise variance) is well known in wavelet literature as the Universal threshold.
- It is the optimal threshold in the asymptotic sense and minimizes the cost function of the difference between the function and the soft thresholded version of the same in the L2 norm sense.
- It is useful for obtain a starting value when nothing is known of the signal condition.

VisuShrink

- VisuShrink is thresholding by applying the Universal threshold proposed by Donoho and Johnstone.
- This threshold is given by

$$\sigma\sqrt{2\log M}$$

  where σ is the noise variance and M is the number of pixels in the image.
- For denoising images, VisuShrink is found to yield an overly smoothed estimate.

SURE Shrink

- SUREShrink is a thresholding by applying subband adaptive threshold.
- It is based on Stein's Unbiased Estimator for Risk (SURE), a method for estimating the loss in an unbiased fashion.
- Let wavelet coefficients in the jth subband be *{ Xi : i* =1,…,d }
- For the soft threshold estimator

$$\hat{X}_i = \eta_t(X_i)$$

  we have

$$SURE(t;X) = d - 2\#\left\{i : |X_i| \le t\right\} + \sum_{i=1}^{d} \min\left(|X_i|, t\right)^2$$

- Select threshold t^S by

$$t^S = \arg\min SURE(t;X)$$

Bayes Shrink

- BayesShrink is an adaptive data-driven threshold for image denoising via wavelet soft-thresholding.
- We assume generalized Gaussian distribution (GGD) for the wavelet coefficients in each detail subband.
- We then try to find the threshold T which minimizes the Bayesian Risk.

Comparison based on minimum MSE